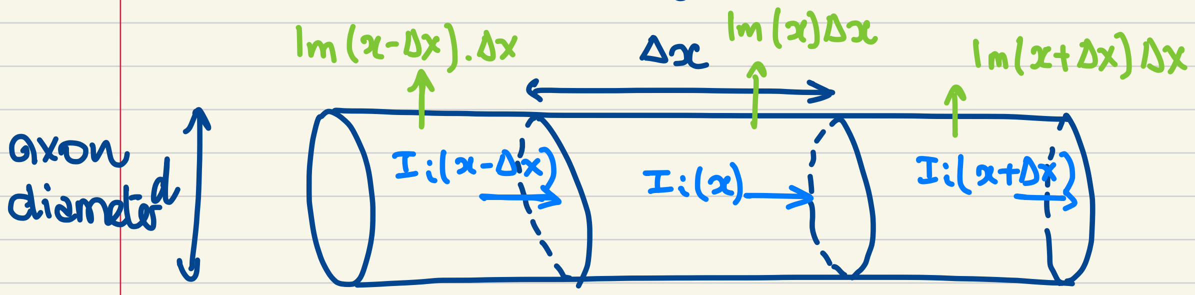
NX-422
Fall 2022
SP LACOUR

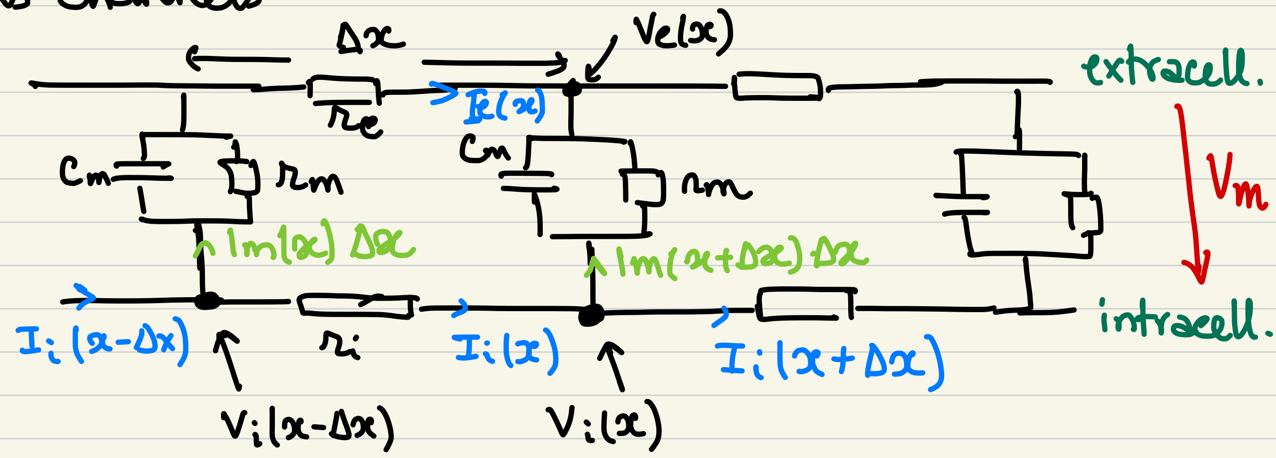
Cable theory

axon model as a cylinder (diameter d)



Axial current flows from one subcylinder into the next one, through a resistance $r_i \, \Delta x$.

Membrane current flows through the parallel combination of the membrane capacitance $c_m \Delta x$ and membrane ions channels



Ohm's law : $V_i(x) - V_i(x+\Delta x) = I_i(x)\, r_i\, \Delta x$

$V_e(x) - V_e(x+\Delta x) = I_e(x)\, r_e\, \Delta x$

when $\Delta x \to 0 \Rightarrow$
$$\begin{cases} \dfrac{\partial V_i(x)}{\partial x} = -\, r_i\, I_i(x) \\[2ex] \dfrac{\partial V_e(x)}{\partial x} = -\, r_e\, I_e(x) \end{cases} \qquad (1)$$

At a given node : $\sum i = 0$
(Kirchoff's law)

$$I_i(x-\Delta x) - I_i(x) = I_m(x)\,\Delta x$$

$$\Rightarrow \frac{\partial I_i(x)}{\partial x} = -\, I_m(x) \qquad (2)$$

and $$\frac{\partial I_e(x)}{\partial x} = I_m(x)$$

Membrane current : $I_m(x)\,\Delta x$
(current across the RC element & channels)

$$I_m(x)\,\Delta x = I_{ion}(x, V, t)\,\Delta x + C_m\,\Delta x\,\frac{\partial V}{\partial t} \quad (3)$$

(1) & (2) $\Rightarrow$ $V_m = V_i - V_e$

$$\frac{\partial^2 V_m}{\partial x^2} = \frac{\partial^2 (V_i - V_e)}{\partial x^2}$$

$$= -r_i\,\frac{\partial I_i}{\partial x} + r_e\,\frac{\partial I_e}{\partial x}$$

$$\frac{\partial^2 V_m}{\partial x^2} = (r_i + r_e)\,I_m$$

(3) $\Rightarrow$ $$\frac{1}{r_i + r_e}\,\frac{\partial^2 V_m}{\partial x^2} = c_m\,\frac{\partial V_m}{\partial t} + I_{ion}$$

$I_{ion} = g_m V_m$) current across the membrane (with channels)

$$\Rightarrow \frac{1}{r_i + r_e}\,\frac{\partial^2 V_m}{\partial x^2} = c_m\,\frac{\partial V_m}{\partial t} + \underbrace{g_m}_{= g_{Na} + g_K + g_{Cl} = \frac{1}{r_m}} V_m$$

$$\underbrace{\boxed{\frac{r_m}{r_i + r_e}}}_{= \lambda^2 \text{ length constant}}\,\frac{\partial^2 V_m}{\partial x^2} = \underbrace{\boxed{r_m\,c_m}}_{= \tau_m \text{ time constant}}\,\frac{\partial V_m}{\partial t} + V_m$$

$$\lambda = \sqrt{\frac{r_m}{r_i + r_e}} \qquad \tau_m = r_m\,c_m$$

Parameters

$\boxed{r_i}$: internal resistance of the axon

$$r_i = \frac{R_i}{\pi d^2} \; / \; \frac{[\Omega.cm]}{[cm^2]}$$

resistance per unit length
Ω/cm

$R_i \sim 60-200 \ \Omega.cm$

r_e : extracellular resistance
Ω/cm
compared to the axon r_i, $\left\{\begin{matrix} r_e \\ r_m \end{matrix}\right.$ is negligeable

$\boxed{c_m}$: capacitance of the membrane
$\mu F/cm$

$$c_m = 2\pi d \, C_m$$

$$C_m \sim 1 \ \mu F/cm^2$$

$$\boxed{g_m} = \frac{1}{r_m} = \frac{2\pi d}{R_m} \quad S/cm$$

$$R_m = 10^4 - 10^5 \ \Omega.cm^2$$

length constant : $\lambda = \sqrt{\frac{r_m}{r_e + r_i}} \sim \sqrt{\frac{r_m}{r_i}} = \sqrt{\frac{R_m d}{2R_i}}$

$$\boxed{\lambda \propto \sqrt{d}}$$

function of axon diameter

time constant : $\boxed{\tau_m = R_m C_m}$

indep. on axon diameter